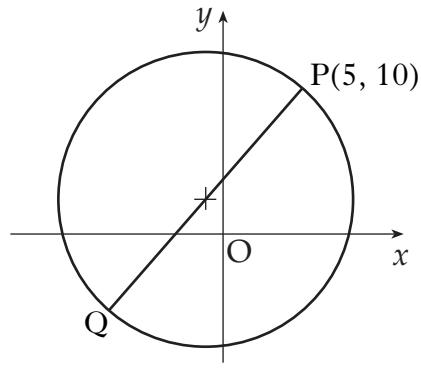
circles problems

[SQA] 1.

(*a*) Show that the point $P(5, 10)$ lies on circle C_1 with equation $(x+1)^2 + (y-2)^2 = 100$. **1**

(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**

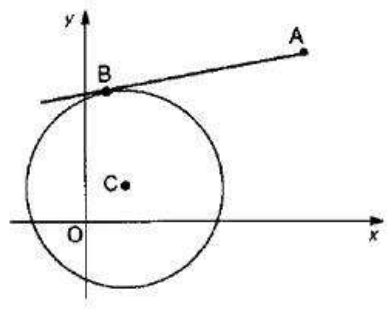


(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q.

The radius of each of these circles is twice the radius of circle C_1.

Find the equations of circles C_2 and C_3. **4**

[SQA] 2. AB is a tangent at B to the circle with centre C and equation $(x-2)^2 + (y-2)^2 = 25$.
The point A has co-ordinates (10, 8).
Find the area of triangle ABC. **5**



3. Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.

The circles have no points of contact.

What is the range of values of p? **9**

[SQA] 4. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

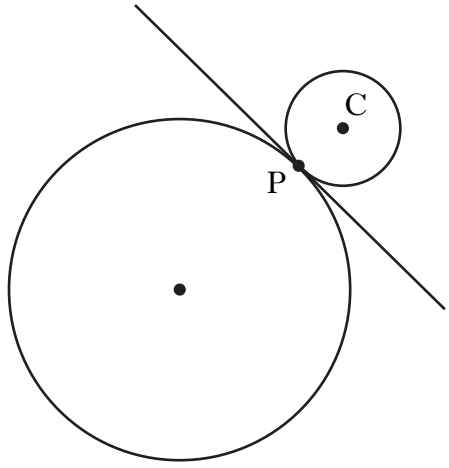
(*b*) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

5. (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the points of contact, P. **5**

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



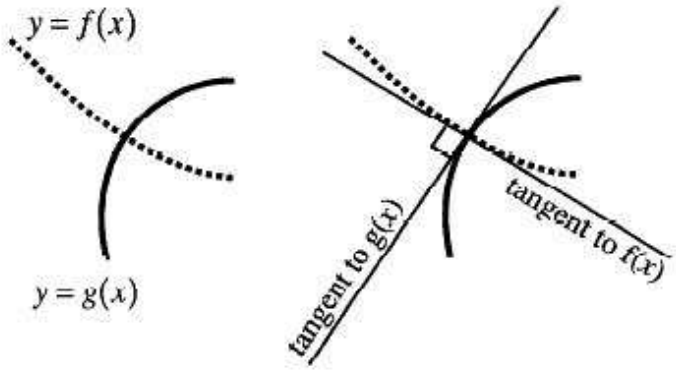
The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

[SQA] 6.

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



(*a*) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y-5)^2 = 13$.
These two curves intersect at (3, 7) and (–3, 7).
Prove that these curves are orthogonal. **(6)**

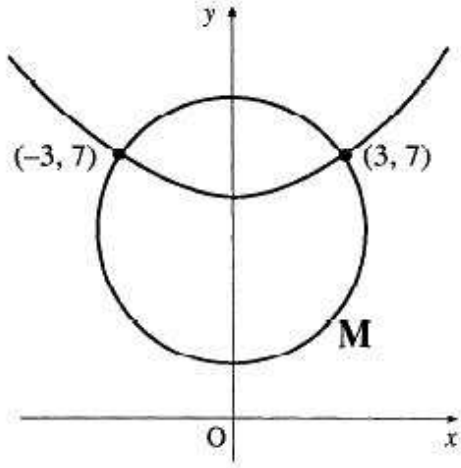


Diagram 1

(*b*) Diagram 2 shows the circle M, from (*a*) above, which is orthogonal to the circle N. The circles intersect at (3, 7) and (–3, 7).

(i) Write down the equation of the tangent to circle M at the point (–3, 7). **(1)**

(ii) Hence find the equation of circle N. **(3)**

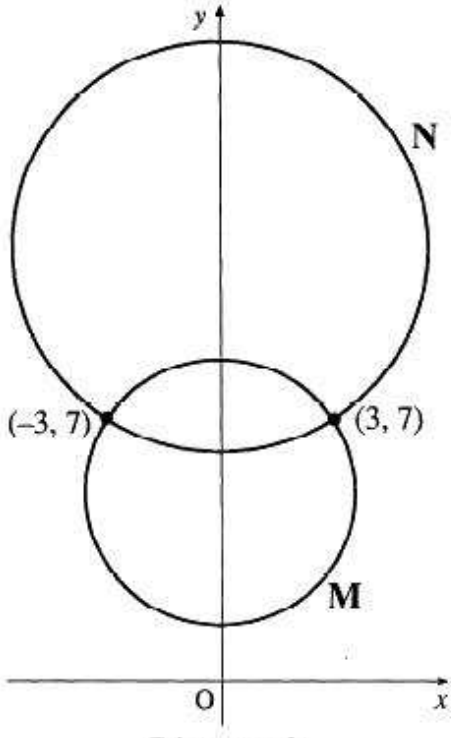


Diagram 2